Journal of Mathematics Education at Teachers College

Fall – Winter 2010

A Century of Leadership in Mathematics and its Teaching

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The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

This issue honors Dr. Alexander P. Karp, an Associate Professor in the Program in Mathematics at Teachers College. A native of St. Petersburg, Russia who is the author of more than one hundred publications including textbooks used throughout Russia, Professor Karp represents Teachers College at meetings and conferences throughout the world as well as through his role as managing editor of the *International Journal for the History of Mathematic Education*.

Former Teachers College Professor and Mathematics Education Chair, Howard Franklin Fehr, was among the most influential mathematics educators of his era. Through his many international contacts, he was the organizer of conferences, projects, and publications including the Congresses of Mathematics Education, a seminal conference on Needed Research in the field, and curriculum initiatives including the Secondary School Mathematics Curriculum Improvement Study.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. Themes planned for the 2011 issues are: *Mathematics Curriculum* and *Technology*. *JMETC* readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,000-2,500 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

Library of Congress Cataloging-in-Publication Data

Journal of mathematics education at Teachers College
p. cm.
Includes bibliographical references.
ISSN 2156-1397
EISSN 2156-1400
1. Mathematics—Study and teaching—United States—Periodicals
QA11.A1 J963

More Information is available online: www.tc.edu/jmetc

Call for Papers

The "theme" of the spring issue of the *Journal of Mathematics Education at Teachers College* will be *Mathematics Curriculum*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2000-2500 words describing research, experiments, projects, innovations, or practices related to mathematics curriculum. Articles should be submitted to Ms. Krystle Hecker at jmetc@tc.edu by January 1, 2011. The spring issue's guest editor, Nicholas Wasserman, will send contributed articles to editorial panels for "blind review." Reviews will be completed by February 1, 2011, and final drafts of selected papers are to be submitted by March 1, 2011. Publication is expected in mid-April, 2011.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the spring 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the blind manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at jmetc@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Spring 2011	Curriculum
Fall 2011	Technology
Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids
Fall 2014	Special Students

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Lasse Savola studied mathematics at Rice University (B.A., 1997) and mathematics education at Teachers College Columbia University (M.S., 2000; Ph.D., 2008). He is an assistant professor of mathematics at the Fashion Institute of Technology—SUNY in New York City, where he teaches courses such as Geometry and the Art of Design as well as Statistics. He is interested in patterns of all kinds. A native of Finland, Lasse enjoys playing ice hockey and music.

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Jean W. Richard holds the Ph.D. in Mathematics Education from Teachers College Columbia University. He has been a faculty member at BMCC since the Fall of 2003. Before coming to BMCC, Richard taught mathematics at the State University of Haiti. His main research interests are Riemann surfaces, international and comparative mathematics education, and the history of mathematics. Besides teaching different courses at BMCC, he is a deputy chairperson involved with a range of administrative duties.

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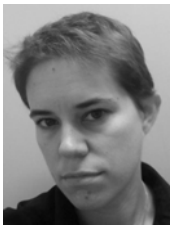

AJ Stachelek earned her M.S. degree in Statistics from the University of Massachusetts–Amherst, AJ taught for the several years in various academic settings ranging from high school to community college. Currently, she is pursuing her Ed.D. in Mathematics Education at Teachers College Columbia University. After teaching in a gifted high school for two years, she discovered her primary research interest in the intersection of gifted education and gender equity in mathematics education.

Comparison of the Classroom Practices of Finnish and Icelandic Mathematics Teachers

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ABSTRACT: Mathematics teachers in Finland and Iceland are on different tracks. Based on a recent video study, the classroom practices of Finnish mathematics teachers seem to be rather traditional. This is in contrast to the Icelandic teachers many of whom use progressive-minded, learner-based instructional strategies. The classroom practices in Finland include substantial whole-class interaction, while many students in Iceland are getting used to learning independently, without significant collaboration with others. This paper discusses three pedagogical dimensions on which mathematics teaching in Finland and Iceland differ: *individualization*, *learner control*, and *content-related public discourse*.

Introduction

This article presents ideas that stem from a recent video-based study comparing the classroom practices of Finnish and Icelandic mathematics teachers. The structures of forty lessons—two lessons from ten randomly selected mathematics teachers of 14 and 15-year-olds in each country—were analyzed as part of a dissertation study (Savola, 2008)[1]. These two Nordic countries were chosen in part because of their performance in the PISA studies; Finnish schools have excelled in each of the PISA assessments thus far, while Iceland has faced lower, declining scores. Iceland also has been the only country where the girls have significantly outscored the boys in mathematics (OECD, 2004, 2007).

It is natural to ask whether something can be learned from the Finnish teachers. Perhaps some of the practices they employ and the norms they set within their classrooms could be effective in Iceland as well as in other countries. On the other hand, Finnish teachers

[1] See (Savola, 2010) for a more concise report of the video study.

can also benefit from learning about the classroom practices of others. However, while bridging what is known and done in different countries—and having everyone gain from the process—it is important to keep in mind that there is no "one-size-fits-all" educational system or strategy; what works in Finland will not necessarily work anywhere else. Cultures, educational and other, are "situated contextual organisms" (Goldman, 2007, p. 33) that have the ability to adapt and morph only within certain limits.

Despite the modest sample size of the video study, some national patterns and cross-national differences can be detected. All twenty Finnish lessons as well as nine of the Icelandic lessons essentially follow the conventional *Review-Lesson-Practice* [RLP]-script, which dates back to at least 1835 when Herbart postulated a cyclical sequence of learning steps (Dunkel, 1969). Eleven of the Icelandic lessons exhibit versions of *Individualized learning* [IL], a constructivist pedagogical philosophy. Content-related whole-class discussions can be missing entirely from the IL-lessons; instead, the teacher tutors the students one-on-one. This is in contrast with the Finnish lessons where teacher-lead activities, which often involve student participation, are emphasized.

Individualization, *learner control*, and *content-related public discourse* are three of the pedagogical dimensions on which the Finnish and Icelandic mathematics classroom practices seem to differ. These dimensions are profoundly related to one another.

Individualization

Mathematics teachers in Finland and Iceland have different roles in the classroom. At the risk of gross overgeneralization, it could be said that Finnish mathematics classrooms are teacher-centered with the teacher being the "sage on the stage," the deliverer of knowledge. Conversely, in many Icelandic classrooms the learning environment is student-centered, and the teacher's role is more that of a "guide on the side." The Finnish approach appears rather conventional, while the Icelandic teachers have ventured onto a more progressive path.

"*Einstaklingsmiðað nám"*—or *Individualized learning* [IL]—has been promoted by the Icelandic educational authorities in recent years (see, e.g., Sigurgeirsson, 2003). As a result, many teachers have adopted a more student-centered approach to their teaching. An integral part of the IL-strategy is to differentiate instruction according to the students' learning needs. Tomlinson (1999) suggests that teachers use, for instance, differentiated curricular materials and tiered activities to accommodate for the dissimilar needs of the students. In a *differentiated classroom*, the content, the learning process, as well as the

attained curriculum—what the students know and can do at the end of the learning process—vary for each student. In such a classroom, the teacher responds to the learning style, instructional needs, interests, and readiness of each unique learner (Tomlinson, 1999). This certainly seems like a logical approach to dealing with typical mixed-ability classrooms. But, based on the video evidence, Tomlinson's ideas have been only partially implemented in the Icelandic schools. The reality is that all the students are usually taught using the same instructional methods and learning materials; they are only moving ahead in the book at somewhat different speeds. In a nutshell, the Icelandic teachers are not doing the preparation that IL requires. This is not surprising as finding suitable materials, teaching methods, and ways of assessment for each student would take lots of time and effort, especially for larger classes. Since the mean class size in the Icelandic lower secondary schools is not small, 19.9 (OECD, 2009, Table D2.1), IL may simply be too tall an order there at this time.

Data from the video study show that the Icelandic teachers spend more time assisting students one-on-one, while there is less seatwork and more whole-class activity in the Finnish mathematics classrooms. This is hardly surprising given the current Icelandic pedagogical climate. The average percentage of class time devoted to assisting individual students or small groups during seatwork is 32.8% for the Finnish lessons in the sample and 55.9% for the Icelandic lessons. Furthermore, in only two of the Finnish lessons did segments of individual assistance take up more than half of the class time, while this was the case in fourteen of the twenty Icelandic lessons. Figure 1 shows the percentages of class time devoted to assisting individual students or small groups.

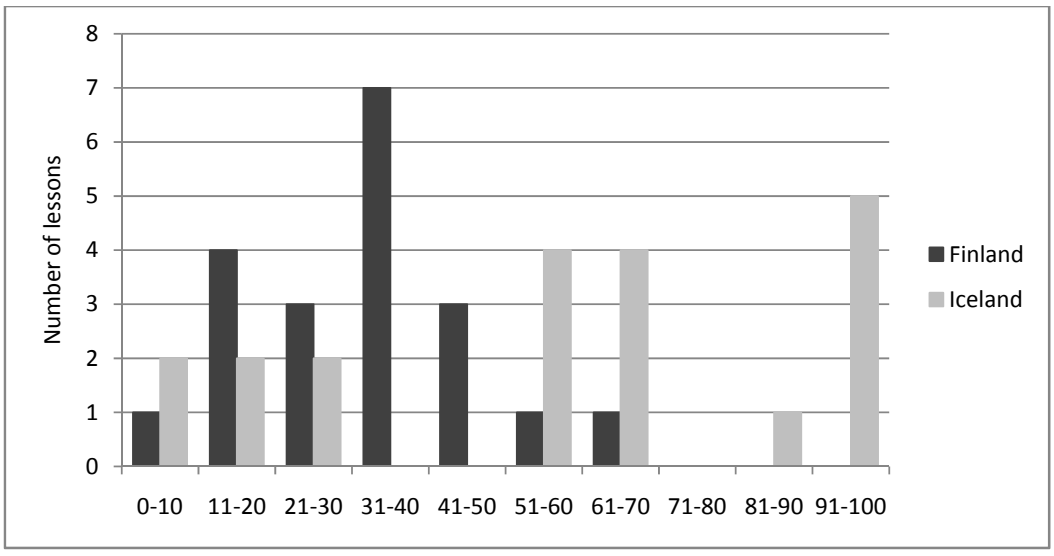

Figure 1: Percentage of class time devoted to assisting students during seatwork

How teachers share their time during seatwork varies. The Finnish mathematics teachers generally are systematic and fair in their treatment of students. During seatwork several Finnish teachers made a point to interact with each student in the class and not just those who asked for help. In contrast, the individual assistance was less evenly distributed in the Icelandic classrooms resulting in some students having no interaction whatsoever with the teacher. It is not clear how the Icelandic teachers decide who to help during seatwork, although some teachers mentioned that most of their time goes to assisting the slower learners. Although these teachers devote a lot of time to seatwork, they may be failing to adequately facilitate the progress of many of their students.

In the recorded lessons there was only little student-to-student interaction around the content in either country. Although this was not the focus of this study, it seems that having students sitting in groups does not necessarily increase content-related student-to-student interaction. This supports previous research that emphasizes the importance of planning appropriate activities in order to make cooperative learning effective (see, e.g., Good, Mulryan, & McCaslin, 1992; Webb, 1991). There was a prepared assignment for groups to complete in only one lesson, a Finnish one. Also, it is worth noting that in seventeen of the twenty Finnish classrooms the students sat in pairs, while this was done in only seven Icelandic classes. In most of the other Icelandic classrooms the students sat in clusters of four or more desks.

Teaching is about building relationships—between the teacher and the students as well as among students—around the content. Since the Finnish teachers interact with their students typically as a collective and the Icelandic teachers do so more on an individual basis, the relationships between the participants are necessarily different in the two countries. Simola (2005) remarks that Finnish teachers, especially more experienced subject teachers in the lower and upper secondary schools, tend to keep a certain professional distance from the students as well as their families. This is unlike in the other Nordic countries, where many teachers attempt to achieve more intimate and personal relations with their students (Simola, 2005). The video study lends support to this notion. But what is the difference for the learner? What kinds of skills are there to be learned through these different approaches: the business-like aloofness of the Finnish teachers and the more individualized tutoring approach of the Icelandic teachers?

Learner control

The locus of control over the learning process is central to analyzing learning environments. Finnish and Icelandic mathematics teachers manage their classrooms differently in this respect. The Finnish teachers tend to maintain firm control of the learning activities in their classrooms. In contrast, many Icelandic teachers assert less control over their students, who have more autonomy of their own learning process.

Based on the video evidence, the Finnish mathematics teachers typically control the learning experience concurrently for the whole class. Critics would say that the process is insensitive to the learning needs of individual students. Although their manners are pleasant, Finnish teachers can be rather demanding when directing their students' actions. They may tell their students, for instance, when to open their books, when to write something down, and when to interact with each other. The Finnish students generally are respectful toward the teacher and do as they are told. The teachers clearly have authority over their classes, unlike in Iceland, where many teachers struggle to maintain disciplinary control.

The Icelandic teachers assume less control over their students' learning. Seven of the ten Icelandic teachers conducted at least one of their two recorded lessons using IL, which inevitably gives more control to the learner. In these lessons the students are able to move ahead in the curriculum at their own pace and, ideally, work with learning materials that suit their needs. Furthermore, it is common that the students in the IL-classes set their own learning goals and homework load in consultation with the teacher.

Based on the video analysis, both the *social* and *sociomathematical norms* (Yackel & Cobb, 1996) differ considerably between typical learning environments in these two countries; the norms are stricter and more clearly defined in the Finnish classrooms. Social norms—the explicit and implicit obligations and expectations that structure the participants' interaction in a given classroom—structure the shared learning experience and shape the ways in which the teacher and the students participate in the learning process. Norms regarding discourse, for instance, are quite different in Finnish and Icelandic classrooms. For example, during a Finnish lesson it is generally not acceptable to start speaking when someone else is speaking or to speak about matters not related to the content. On the other hand, many Icelandic classrooms seem to lack strict norms for discourse. Thus the platforms for small-group and whole-class discussions are less structured there.

The sociomathematical norms—what constitutes mathematics and doing mathematics—are also different. Many of the Finnish teachers that participated in this study

tend to ask for more than recall of facts or a quick calculation. They often demand justification in addition to simple answers and expect their students to communicate using proper terminology and full sentences. By doing so, they give their students valuable opportunities to explain their thinking. Words like "why" and "explain" are often heard from these teachers. This is a sign of strict sociomathematical norms. This was not the case in the Icelandic classrooms, where the I-R-E discourse pattern (from teacher-*initiated* question, student *response,* and teacher *evaluation*) (e.g., Hiebert & Grouws, 2007) is typical.

The role of homework is different in Finland and Iceland. The Finnish students typically have a homework assignment for every mathematics class. All the students get the same assignment and are held accountable for its completion. Many teachers in the video study checked the students' notebooks to make sure the assignment had indeed been completed. Some of the Finnish teachers suggested extra work to the faster students who had finished the assignment already during seatwork. As previously mentioned, the Icelandic students have recently been given more control over their own homework load. Consequently they are now spending less time doing mathematics homework than before. According to data obtained from the Icelandic Educational Testing Institute (A. Halldórsson, personal communication, July 16, 2008), the changes in the mathematics homework levels from 2003 to 2006 are considerable. For instance, the percentage of 15-year-olds who spend less than two hours doing mathematics homework weekly increased from 45.1% in 2003 to 67.1% in 2006, and the percentage of students who spend six or more hours per week at home doing mathematics dropped from 16.2% to a mere 1.6% in those three years.

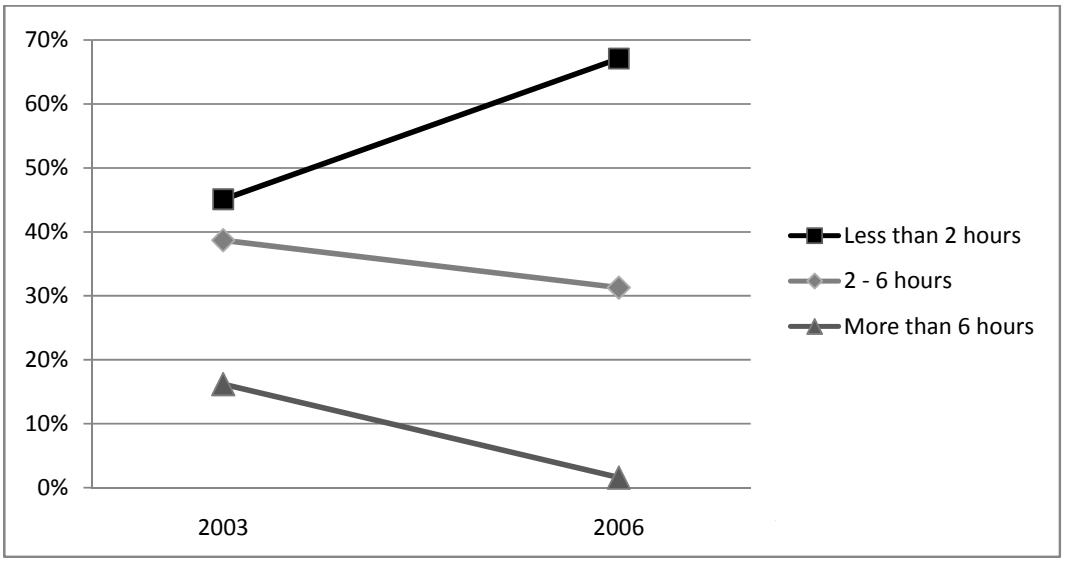

Figure 2: Time spent on mathematics homework weekly by Icelandic 15-year-olds in 2003 and 2006

These changes in homework levels correspond to a 9-point drop—from 515 to 506—in Iceland's mathematics scores in PISA from 2003 to 2006 (OECD, 2007). It is likely that the increasing freedom from constraints at school and the lack of extrinsic motivation to do mathematics at home have contributed to the recent declines in mathematics attainment for Icelandic students. The role of homework in the Icelandic curriculum warrants further research.

Content-related public discourse

Content-related classroom interaction yields one of the main differences between the practices of the Finnish and Icelandic mathematics teachers. The Finnish lessons in the sample include many segments during which the teacher stimulates whole-class discussions about the topic at hand. Also, Finnish students regularly are asked to present their solutions in front of the class. These lesson elements are not as common in the Icelandic *Review-Lesson-Practice* [RLP]-lessons, and they are almost entirely missing from the IL-lessons. Overall, Finnish mathematics teachers in the video study provided more opportunities for learning through classroom interaction and the use of language than their Icelandic counterparts.

Social interaction around content and the use of language in the classroom facilitate the students' constitution of mathematical meanings (Cobb & Bauersfeld, 1995; Lave & Wenger, 1991). Current research suggests that in order to make sense of mathematical concepts, students need opportunities to share their mathematical thinking, make conjectures, discuss alternative approaches to problem-solving, and so on (see, e.g., Franke, Kazemi, & Battey, 2007). Learning mathematics is not a one-way street; it is not simply receiving information. It is about participating in a community whose members negotiate meaning together by communicating with each other using mathematical language. And, naturally, the more that community engages in discourse about mathematics, the better its members learn to communicate in mathematical language. This in turn promotes learning mathematics.

Communicating in mathematical language is especially important in developing conceptual understanding. According to Brophy (1999), effective teaching of mathematics is centered around content-related discourse, and

> [the] discourse is not limited to rapidly paced recitation that elicits short answers to miscellaneous questions. Instead it features sustained and thoughtful development of

> key ideas. Through participation in such discourse, students construct and communicate content-related understandings. In the process, they abandon naïve ideas or misconceptions and adopt the more sophisticated and valid ideas embedded in the instructional goals (1999, p.19).

Some of the Finnish mathematics teachers in the video study provided nice examples of how to lead thoughtful classroom discourse. As mentioned in the previous section, these teachers would often press their students to justify and elaborate on their answers, thereby promoting strict sociomathematical classroom norms. By doing so, they generate more learning opportunities for their students.

The Finnish teachers elicited classroom interaction especially during lesson segments dedicated to reviewing material from previous lessons. In the video study, the forty Finnish and Icelandic lessons were coded using four functional categories: *Review*, *Introducing New Content*, *Practicing/Applying*, and *Other*. The various lesson segments were then analyzed with the focus on the form of social interaction. For example, the *Review*-segments were further divided into three categories: 1) *Teacher discusses examples or a concept in the front*, 2) *Students present solutions on the board*, and 3) *Class works together on a problem*. In the sample, 50.1% of the *Review*-segments within the Finnish lessons can be classified as having the form "*Teacher discusses examples or a concept in the front*." The students were more actively involved by working on the board or participating in a discussion in the other half of the *Review*-segments. Some teachers would ask every student to contribute to the discussion. In contrast, the eleven Icelandic IL-lessons had no public review and, in the nine RLP-lessons, 85.5% of the time spent reviewing essentially featured a teacher monologue. Notably no student presentations took place during the Icelandic RLP-lessons[2].

Some learner-based instructional methods, such as *problem-based learning* and *inquiry learning*, promote social interaction and the use of language through collaborations (e.g., Hmelo-Silver, Duncan, & Chinn, 2007). In the Icelandic mathematics classrooms it is common for the students to sit in clusters. This, however, does not ensure any of the benefits that collaboration can yield. As Good, Mulryan, and McCaslin (1992) point out, all forms of collaborative learning are not equally effective. Careful planning of the learning activities and the appropriateness of the tasks are fundamental to success, and students should be attentively guided through the collaborative process to keep the discussion focused on the targeted skills. Furthermore, Slavin (1983) notes that group study without group rewards and a strong sense of individual accountability is not linked with increased student achievement.

[2] For more details, see (Savola, 2008) or (Savola, 2010).

Conclusion

Mathematics teaching in Finland and Iceland differ in a number of ways, some of which have been discussed in this article. A recent video study suggests that "the Finnish way" places the teacher at front and center of the classroom while the students participate in the interactive learning experience. On the other hand, the Icelandic educational system is going through a transitional phase as the student-centered IL is replacing some of the more traditional instructional methods. Many Icelandic teachers foster individualization and learner control, but unfortunately this comes at the cost of content-related public discourse. Finnish teachers have maintained control of their students' learning experiences and often lead their classes into meaningful discussions about mathematics.

Although nowadays the Finnish schools allow for more individualization and curricular choice for students as opposed to twenty years ago (Välijärvi et al., 2007), Finnish teachers still teach to the whole class. The classes are heterogeneous; there is no tracking in the Finnish school system. Many mathematics teachers make sure everyone does their work and participates in the lesson, especially during review. The participatory nature of the lessons and, more specifically, the presence of content-related public discourse, may be a factor in the recent educational successes of the Finns.

The premise of the Finnish system is educational equity. The old wisdom has been that a small and remote country like Finland cannot afford not to offer everyone a high quality education (Välijärvi et al., 2007). Same goes for many other social services. Consequently the Finns have faith that the system will take care of an individual, a "if you just do as you're told, you'll be fine"-mentality. This bodes well for the Finnish educators who get to teach rather obedient students. They also generally get the support of the parents. In fact, perhaps the most important reason behind the Finnish success story is the respect that the teaching profession enjoys within the society. These cultural characteristics—the innate obedience and the relatively high esteem of teachers—have helped propel Finnish schools to the top.

If the Nobel Price-winning book *Independent People* by Halldór Laxness (1997, original issue in 1946) is any indication, Icelanders have a deep-seated distrust of the establishment. The main character of the book is Bjartur, a farmer. Despite living in harsh conditions and being plagued by superior forces, Bjartur refuses help from others and, instead, puts up a futile fight to gain absolute independence. Like the resilient farmer, Iceland has always cherished its independence. For over a thousand years it has been a defiant

cultural and political outpost of Europe. Given these fundamental differences in the cultural characteristics of Finns and Icelanders, it is no wonder that their school systems and instructional methods are different.

But perhaps the students in the IL-classes are learning different, yet equally important skills like time management and self-motivation? Maybe Iceland simply needs some time to get through the transition. Maybe. But to make IL work in any classroom, the teacher needs to carefully plan the activities and use differentiated materials to suit the varying needs of the students. Part of the problem is that currently the Icelandic lower secondary school mathematics teachers, for instance, have only two Icelandic book series (*Almenna Stærfræði* and *Átta-tíu*[3]) from which to choose. More mathematical material written in Icelandic is needed.

For the students IL means that they can work through the book at their own pace. But what is the benefit of finishing the curriculum early? If the goal of IL is to keep the faster students from getting bored, perhaps it was forgotten that mathematics is not just a sequence of chapters and books. Learning mathematics should not be like racing from one end of a railroad track to another, with no detours. Gifted students certainly would benefit from additional work that expands their view of mathematics laterally. Perhaps doing interesting projects and different types of mathematics than is normally done in the school curriculum could ignite a joy of mathematics and lead to further study.

No article on the Icelandic school system would be complete without some discussion about gender. In PISA 2003, Iceland was the only country where the girls significantly outperformed the boys in mathematics. According to Halldórsson and Ólafsson (2009), the Icelandic girls have held an educational advantage over the boys in recent years; however, 2003 was somewhat exceptional as the gender difference typically has not been as large as it was then. Halldórsson and Ólafsson conclude from national tests conducted each year to 9, 12, and 15-year-olds as well as grade records from Icelandic universities that the girls' advantage is apparent in most subjects from elementary school through the university level. The gender difference has remained relatively stable over the past years. Also, the difference is about the same for each performance level: the low, the medium, and the high performing students. Thus it seems safe to say that the PISA 2003 results were not a fluke. But why is this so? One possible explanation is the so-called "Jokkmokk effect," the idea that boys in rural areas of Iceland value traditional careers such as fishing over academics and that the

[3] The latter is promoted by the Icelandic educational administration.

best option for girls is to seek education in the colleges and universities. However, this explanation has been ruled out by researchers (see, e.g., Ólafsson, Halldórsson, & Björnsson, 2006). Instead, differing attitudes towards school and the classroom culture are more likely to explain at least some of the Icelandic gender gap. Doing well academically is a more personal issue for girls while the boys' well-being is more related to other activities such as sports or being "cool." Social pressure may also be a significant factor. It is telling that for Icelandic boys the most important variable affecting academic performance is perceived discipline in the classroom, while the girls' school performance can be linked to anxiety, self-perception, and motivation (Halldórsson & Ólafsson, 2009).

References

Brophy, J. (1999). *Teaching* (Educational Practices Series No. 1). Geneva, Switzerland: International Bureau of Education. Retrieved July 22, 2010 from http://www.ibe.unesco.org/fileadmin/user_upload/archive/publications/EducationalPracticesSeriesPdf/prac01e.pdf.

Cobb, P., & Bauersfeld, H. (Eds.). (1995). *The emergence of mathematical meaning: Interaction in classroom cultures*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Dunkel, H. B. (1969). *Herbart and education*. New York: Random House.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256). NCTM. Charlotte, NC.

Goldman, R. (2007). Video representations and the perspectivity framework: Epistemology, ethnography, evaluation, and ethics. In R. Goldman, R. D. Pea, B. Barron & S. J. Derry (Eds.), *Video research in the learning sciences* (pp. 3-37). Mahwah, NJ: Lawrence Erlbaum Associates.

Good, T., Mulryan, C., & McCaslin, M. (1992). Grouping for instruction in mathematics: A call for programmatic research on small-group processes. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp.165-198). NCTM. New York: Macmillan.

Halldórsson, A., & Ólafsson, R. (2009). The case of Iceland in PISA: Girls' educational advantage. *European Educational Research Journal, 8(1),* 34-53.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). NCTM. Charlotte, NC: Information Age Publishing.

Hmelo-Silver, C., Duncan, R. G., & Chinn, C. (2007). Scaffolding and achievement in problem-based and inquiry learning: A response to Kirschner, Sweller, and Clark (2006). *Educational Psychologist, 42*(2), 99-107.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press.

Laxness, H. (1997). *Independent people*. (Reissue). New York: Vintage International.

OECD. (2004). *Learning for tomorrow's world: First results from PISA 2003*. Paris: OECD Publication.

OECD. (2007). *PISA 2006 science competencies for tomorrow's world*. Paris: OECD Publication.

OECD. (2009). *Education at a glance: OECD indicators 2009*. Paris: OECD Publication.

Ólafsson, R. F., Halldórsson, A. M., & Björnsson, J. K. (2006). Gender and the urban-rural differences in mathematics and reading: An overview of PISA 2003 results in Iceland. In J. Mejding & A. Roe (Eds.), *Nordic lights on PISA 2003: A reflection from the Nordic countries* (pp. 185-198). Copenhagen, Denmark: Nordic Council of Ministers.

Savola, L. (2008). *Video-based analysis of mathematics classroom practice: Examples from Finland and Iceland*. (Doctoral dissertation). Columbia University. ProQuest/UMI (AAT 3317607).

Savola, L. (2010). Structures of Finnish and Icelandic mathematics lessons. In B. Sriraman, C. Bergsten, S. Goodchild, G. Palsdottir, B. D. Sondergaard & L. Haapasalo (Eds.), *The first sourcebook on Nordic research in mathematics education*. Charlotte, NC: Information Age Publishing.

Sigurgeirsson, I. (2003). Skref í átt til einstaklingsmiðaðs náms. Retrieved November 14, 2009 from http://starfsfolk.khi.is/ingvar/namskeid/Fraedslumidstod/vefur/.

Simola, H. (2005). The Finnish miracle of PISA: Historical and sociological remarks on teaching and teacher education. *Comparative Education, 41*(4), 455-470.

Slavin, R. (1983). When does cooperative learning increase student achievement? *Psychological Bulletin, 94*, 429-445.

Tomlinson, C. A. (1999). *The differentiated classroom: Responding to the needs of all learners*. Alexandria, VA: ASCD.

Välijärvi, J., Kupari, P., Linnakylä, P., Reinikainen, P., Sulkunen, S., Törnroos, J., & Arffman, I. (2007). *The Finnish success in PISA and some reasons behind it 2*. Jyväskylä, Finland: Institute for Educational Research, University of Jyväskylä.

Webb, N. (1991). Task-related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education*, 22, 366-389.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*, 458-477.